

Inverse Hyperbolic Functions

$$\sinh^{-1}x = \ln(x + \sqrt{x^2 + 1})$$

$$\frac{d}{dx} \sinh^{-1}x = \frac{1}{\sqrt{1+x^2}}$$

$$\cosh^{-1}x = \ln(x + \sqrt{x^2 - 1}) \quad (x \geq 1)$$

$$\frac{d}{dx} \cosh^{-1}x = \frac{1}{\sqrt{x^2 - 1}}$$

$$(x > 1)$$

$$\tanh^{-1}x = \frac{1}{2} \ln \frac{1+x}{1-x} \quad (|x| < 1)$$

$$\frac{d}{dx} \tanh^{-1}x = \frac{1}{1-x^2} \quad (|x| < 1)$$

$$\coth^{-1}x = \frac{1}{2} \ln \frac{1+x}{1-x} \quad (|x| > 1)$$

$$\frac{d}{dx} \coth^{-1}x = \frac{1}{1-x^2} \quad (|x| > 1)$$

$$\operatorname{sech}^{-1}x = \ln \left(\frac{1 + \sqrt{1-x^2}}{x} \right) \quad (0 < x \leq 1)$$

$$\frac{d}{dx} \operatorname{sech}^{-1}x = \frac{-1}{x\sqrt{1-x^2}} \quad (0 < x < 1)$$

$$\operatorname{csch}^{-1}x = \ln \left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|} \right) \quad (x \neq 0)$$

$$\frac{d}{dx} \operatorname{csch}^{-1}x = \frac{-1}{|x|\sqrt{1+x^2}} \quad (x \neq 0)$$

$$\int \frac{1}{\sqrt{1+x^2}} dx = \sinh^{-1}x + C$$

$$\int \frac{1}{\sqrt{x^2-1}} dx = \cosh^{-1}x + C \quad (x > 1)$$

$$\int \frac{1}{1-x^2} dx = \begin{cases} \tanh^{-1}x + C & |x| < 1 \\ \coth^{-1}x + C & |x| > 1 \end{cases} = \frac{1}{2} \ln \left| \frac{1+x}{1-x} \right| + C$$

$$\int \frac{1}{x\sqrt{1-x^2}} dx = -\operatorname{sech}^{-1}|x| + C$$

$$\int \frac{1}{x\sqrt{1+x^2}} dx = -\operatorname{csch}^{-1}|x| + C$$